Terramechanics Tyre Model

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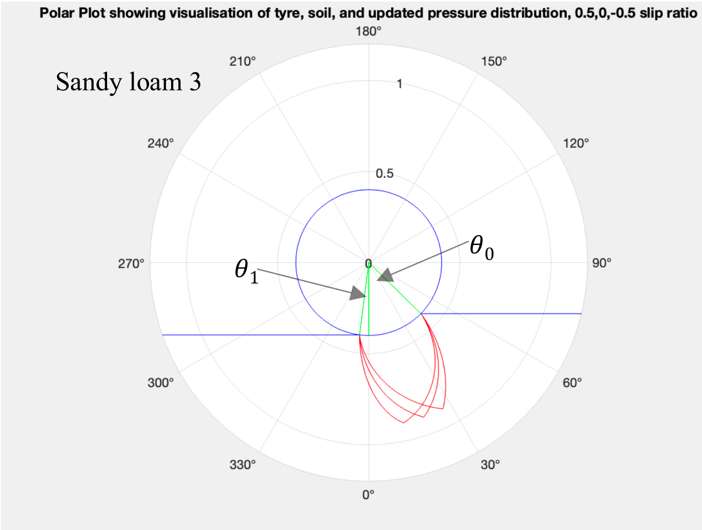


Figure 1, Visualisation of tyre-terrain relationship and changing pressure distribution at varying slip ratios

Terramechanics is defined by (Wong, 2001) as *'the study of the performance of an off-road vehicle in relation to its operating environment (the terrain)'*. This project began with simplified modelling using pressure–sinkage and shear stress equations, applied over the front portion of a circular tyre profile. Basic off-road tyre dimensions and constant or varying slip ratios were used to explore fundamental relationships between terrain and tyre forces. Terrain return due to elastic unloading was introduced, defining an exit angle and the total angle over the contact area. This allowed for calculation of a forward shifted point of maximum pressure, with the degree of shift being a function of slip. From this a new pressure distribution was defined using an equation provided by (REECE, 1967) – an adaptation of Bekker's equation. This pressure distribution was substituted into shear stress equations to model negative and positive portions of tyre tread. This was still under a rigid tyre assumption, so the vertical load was used to iteratively compute the deflection of a pneumatic tyre, producing a deformed contact patch and varying effective radius. Updated vertical load and drawbar pull were calculated across this arc. The tyre model was implemented in a Simulink-compatible function block and placed in a quarter-car suspension model. Vertical and horizontal feedback loops were created and coupled once independent results were verified, allowing dynamic interaction between slip, torque, sinkage, and suspension response. Final outputs and graphs of velocity and sinkage verified expected behaviour and equilibrium vertical load. The work highlights key force interactions while pointing to future extensions including lateral forces, damping, and secondary effects like fuel efficiency and mechanical wear.

1. Introduction

(Wong, Theory of Ground Vehicles , 2001) defines terramechanics as *'the study of the performance of an off-road vehicle in relation to its operating environment (the terrain)'*. This highlights the two main interests of terramechanics – the terrain and off-road vehicles but also explains how terramechanics is the interaction of both these facets.

Talking in context of the terrains (Wong, Theory of Ground Vehicles , 2001) also defines terramechanics as '*Mechanical properties of the terrain and its response to vehicular loading'*. This sets a clear focus of measurable, physical change to the terrains and also gives more insight into the 'interaction' being loading.

A terramechanics tyre model follows these definitions but is solely focused on the tyres of the off-road vehicles. A model highlights simulation of physical dimensions of tyres within a computer software. (Wong, 2001) additionally defined performance as 'ability to accelerate,

to develop drawbar pull, to overcome obstacles and decelerate'. This then sets a direction of specific forces or accelerations either on the tyre, the terrain or the pulling force of the vehicle.

A terramechanics tyre model follows these definitions but is solely focused on the tyres of the off-road vehicles. A model highlights simulation of physical dimensions of tyres within a computer software.

In the project preparation report the plan outlined was to take inspiration from advanced tyre models discovered in the literature review, to introduce more tyre modelling phenomena to the simple tyre model, in order for it to be more accurate. The assumptions made that rendered the simple model inaccurate were rigid tyre behaviour, pressure-sinkage following Bekker's equation, a maximum pressure point at bottom dead centre of tyre, and no return of terrain. Each phenomenon introduced will eliminate one of these assumptions, hence making the model more accurate. Section 2 of this paper will follow this plan, with an explanation of the concept being introduced, the importance of the concept to the accuracy of the model, expected results, an explanation of the method in MATLAB/Simulink, and graphical representation of key results. The order of the explanation of additions to the model, is also the order that the MATLAB script and function inside Simulink follows.

Aims

- Investigate tyre and terrain relationships and how they affect performance factors of an off-road vehicle.
- Investigate desired performance factors of certain industries, developing an understanding of unique requirements and challenges.

Objectives

- Collect a library of terrain data
- Produce a simple tyre model by making assumptions of tyre behaviour that simplify force prediction calculations
- Improve this tyre model by adding more complex tyre behaviour phenomena to generate more accurate force predictions
- Implement improved tyre model into ¼ car model
- Generate lists of challenges faced by industries that employ terramechanics, as well as a list of what unique key forces they are interested in.
- Present advantages and disadvantages of specific modelling techniques for specific requirements.

1.1. Previous work

Based on definitions of terramechanics, terrain relationships to loading is fundamental, so a library of terrain data was generated in MATLAB (shown in Appendix) with data taken from (Wong, 2008), allowing this empirical data to be implemented into the empirical equations that build the tyre model.

(PYTKA, 2010) states that a tyre-terrain model is both the forces acting on the tyre, and stresses acting on the terrain. Listing vertical load, driving force and steering force as the tyre forces, and vertical stress, longitudinal stress, transverse stresses and shear stresses as the terrain stresses. Additionally, the forces that (Wong, Theory of Ground Vehicles , 2001) uses to define motion of a rigid wheel, further explained by (JO-YUNG WONG, 1967) as forces acting on the equilibrium of a wheel of constant velocity on horizontal ground are vertical load, drawbar pull and tractive effort, which are all integrals of components of pressure and shear stress over the contact area of the tyre. Hence, pressure and shear stress were defined as functions of theta, which could be placed within an integral function and multiplied by cos or sin of theta, over the range of angles from zero to the entry angle. This allowed for calculation of vertical load and drawbar pull, with graphs of these against slip ratio produced at varying sinkage values.

The most used pressure equation was Bekker's, with empirical parameters found from the use of a Bevameter, which conducts pressure-sinkage and shear-stress tests.

$$P = \left(\frac{k_c}{b} + k_\phi\right)\zeta^n \tag{1.}$$

Vertical Load:

$$W = rb\left[\int_0^{\theta_0} P(\theta)\cos(\theta)\,d\theta + \int_0^{\theta_0} \tau(\theta)\sin(\theta)\,d\theta\right] \tag{2.}$$

Drawbar Pull:

$$F_d = rb\left[\int_0^{\theta_0} \tau(\theta)\cos(\theta)\,d\theta - \int_0^{\theta_0} P(\theta)\sin(\theta)\,d\theta\right] \tag{3.}$$

An iterative model was produced where a vertical load value was set, and values of sinkage were calculated over varying slip ratios, as this models a set car weight with a varying torque, resulting in a varying sinkage, which is representative of a real-world scenario.

2. Improvements to tyre model

2.1. Introduction of return

2.1.1. Concept

As a tyre moves over a terrain, on the front portion of the tyre the sinkage of each individual point from the entry angle to the bottom dead centre is increasing, loading the terrain. However, after this point the sinkage of each individual point is decreasing, and there is an unloading phase, where depending on the elasticity of the terrain, it will attempt to return to its original form.

2.1.2. Importance to accuracy

Key performance factors of drawbar pull and vertical load are calculated based on the horizontal and vertical components of pressure and shear stress at their individual sinkage's across the contact patch of the tyre and terrain. Therefore, modelling a pressure distribution on the rear portion of the tyre, that allows for the exit angle to be calculated, is needed to calculate the full contact patch of the tyre.

2.1.3. Expected results

Overall increase of vertical load, with vertical component of pressure modelled on the rear supporting vertical load, however the shear stress on the rear portion slightly reduces this increase.

Drawbar pull should be increased as the horizontal component of pressure is acting in the opposite direction (the direction of motion).

2.1.4. Implementation into model

Pressure-sinkage during unloading/reloading. (Wong, Theory of Ground Vehicles (fourth edition), 2008)

$$P = P_u - k_u(z_u - z) \tag{4.}$$

Slope of unloading/reloading line (k_u), degree of elastic rebound. (Wong, Theory of Ground Vehicles (fourth edition), 2008)

$$k_u = k_0 + A_u z_u \tag{5.}$$

k_0 and A_u are empirical parameters that characterise a terrain's response to repetitive loading. Values of these for various terrains are provided by (Wong, Theory of Ground Vehicles (fourth edition), 2008).
P_u and z_u are the pressure and sinkage values at the maximum sinkage point, calculated from the pressure distribution over the front portion of the tyre.

The same range of sinkage values is used to ensure the return point can be calculated, as the return angle won't exceed entry angle. This means the return pressure distribution will range from large values of pressure to negative values of pressure, and the 'min' function is used to calculate the sinkage value where pressure is closest to zero (exit point) and indexed to find the corresponding number within the range of this sinkage value. The exit angle is found by extracting the value within the range of angles at the indexed number. The return pressure distribution is also stopped at this indexed number, to ensure it doesn't display the illegitimate negative numbers.

2.1.5. Results

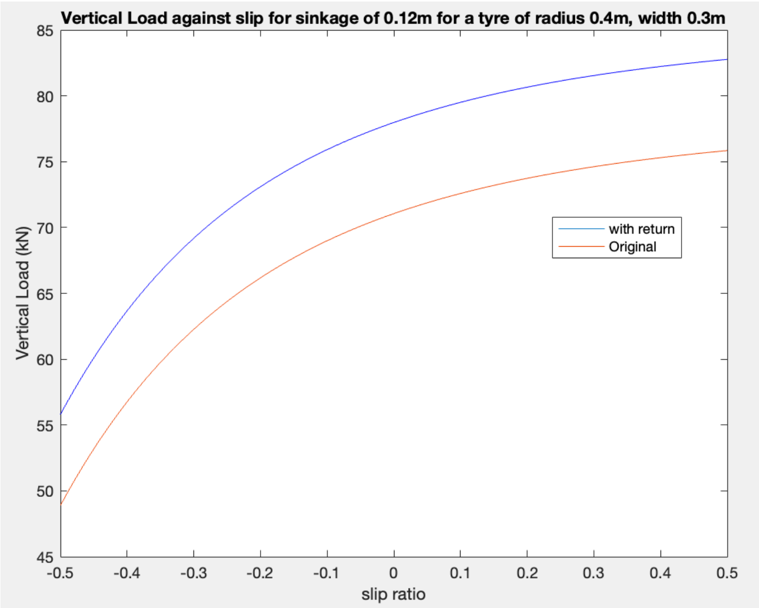


Figure 2, showing vertical load against slip with and without modelling return

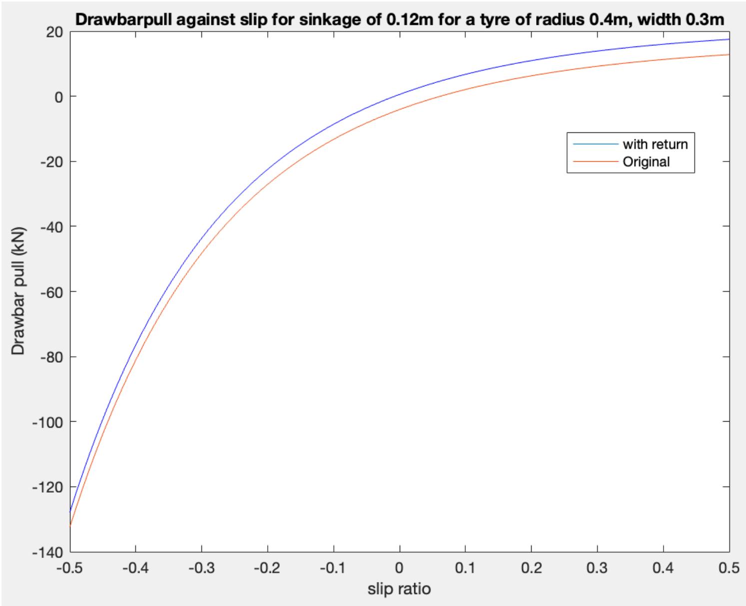


Figure 3,showing drawbar pull against slip with and without modelling return

These results show as expected both vertical load and drawbar pull are increased by modelling return.

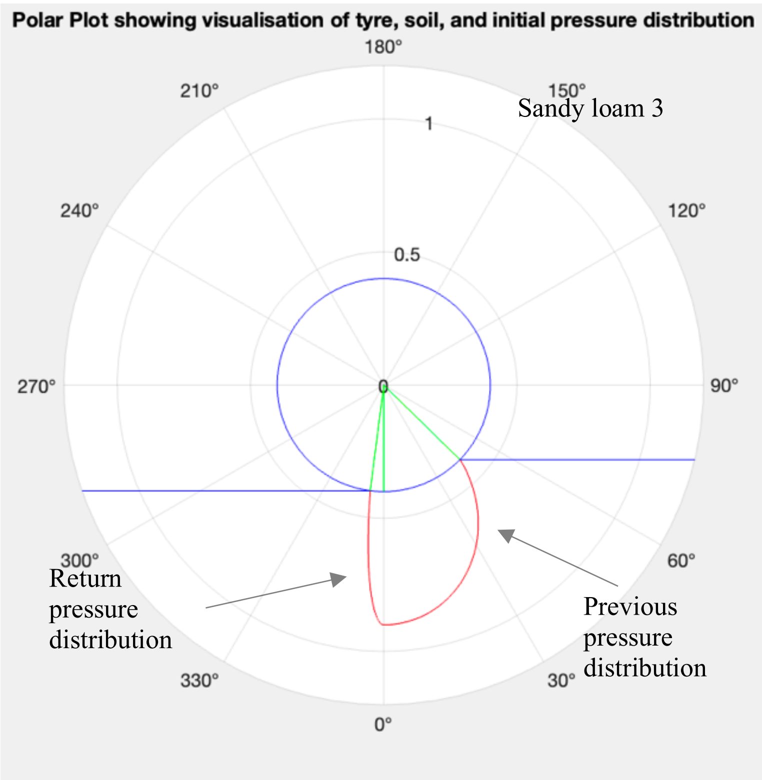


Figure 4, visualising the entry angle, exit angle and additional return pressure distribution

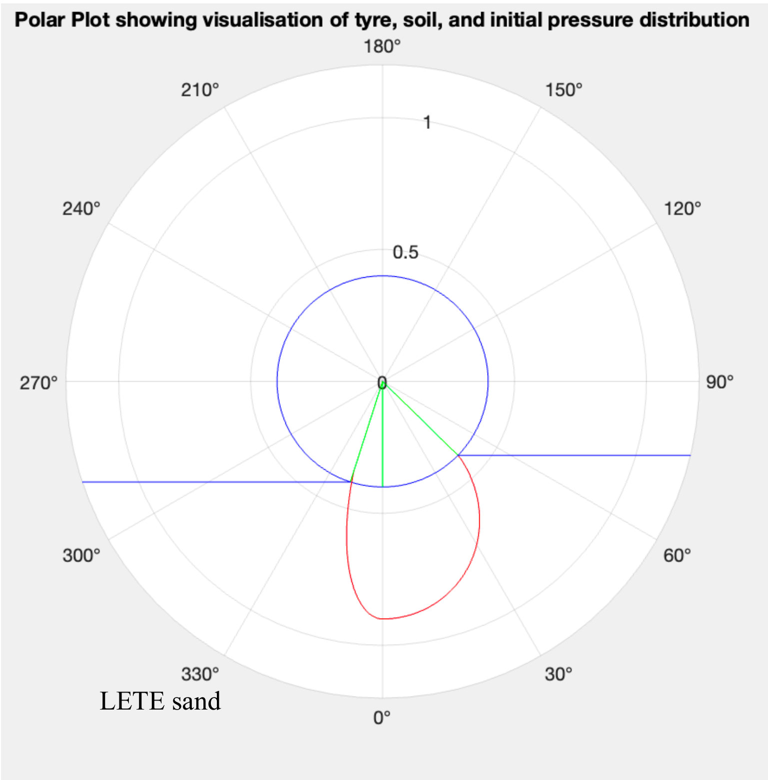


Figure 5, visualising the difference in exit angle for a different terrain

2.2. New pressure distribution with adjusted maximum pressure point

2.2.1. Concept

Based on Bekker's pressure-sinkage equation and visually shown by graphs produced in the preparation report, theoretically the maximum normal pressure point should be at the maximum sinkage (bottom dead centre). (Wong, Theory of Ground Vehicles (fourth edition), 2008) validates this, but states that experimental results show that the maximum normal pressure is in front of this point, and that it also varies with slip ratio. Further stating that the maximum normal pressure occurs at the junction of the two flow zones on the contact patch.

2.2.2. Importance to accuracy

As mentioned above, values of vertical load and drawbar pull are the vertical and horizontal components of pressure and shear stress over the contact patch. As pressure is the larger force to shear stress, both of these performance factors are highly influenced by the value and location of the maximum pressure and the highest pressure values around the maximum point.

A change in the maximum pressure point to a different angle, and therefore a shift in the angles of the largest pressures around the maximum point, changes the ratio of them contributing to vertical load and drawbar pull. For example, where the previous maximum pressure was at bottom dead centre (theta = 0), the pressure would only contribute to vertical load, whereas now the maximum pressure is in front of bottom dead centre (theta = x), it will contribute less to vertical load and more to drawbar pull (negatively).

This also implements (REECE, 1967) adaptation to Bekker's pressure equation, which will increase accuracy as Bekker's pressure equation can cause inaccuracies, which were discussed that in the project preparation report. Bekker's pressure equation can produce accurate results in many cases, but under specific circumstances it starts to become very inaccurate, so for the validity of the model in many scenarios, implementing an adapted pressure equation is important.

2.2.3. Expected results

A decrease in vertical load is expected, however as the pressure that was in the direction of motion in the return phase has been shifted to forwards, it now acts against the direction of motion, so drawbar pull is expected to decrease.

2.2.4. Implementation into model

Maximum pressure point from exit angle:

$$\theta_{m2exit} = [C_1 + (C_2 * slip\ ratio)\ ] * total\ angle \tag{6.}$$

Maximum pressure point from zero (taking exit angle as positive):

$$\theta_{m2zero} = \theta_{m2exit} - \theta_1 \tag{7.}$$

Soil	Soil properties: Angle of internal shearing resistance ϕ	Soil cohesion c (lb/in.²)	Density γ (lb/in.³)	Pressure–sinkage constants: k_1	k_2	n	Shear deformation modulus K (in.)	Coefficients for determining the relative position of maximum radial stress: c_1	c_2
Compact sand	33·3°	0·10	0·0575	20	2·5	0·4706	1·5	0·43	0·32
Loose sand	31·1°	0·12	0·048	0	2	1·1504	1·5	0·18	0·32

Table 1, showing findings of empirical data for C1 and C2 from (REECE, 1967)

Pressure-sinkage equation region in front of maximum pressure point (θ_{m2zero}):

$$P_{front} = \left(\frac{k_c}{b} + k_{phi}\right) * (r^n) * ((\cos(\theta) - \cos(\theta_0))^n \tag{8.}$$

Pressure-sinkage equation region behind maximum pressure point (θ_{m2zero}):

$$P_{rear} = \left(\frac{k_c}{b} + k_{phi}\right) * (r^n) * [\cos\left[\theta_0 - \frac{\theta + \theta_1}{\theta_{m2zero} + \theta_1} * (\theta_0 - \theta_{m2zero})\right] - \cos(\theta_0)]^n \tag{9.}$$

An if/else loop is ran where if the value of theta is greater than or equal to θ_{m2zero}, the front equation contributes to the pressure distribution, and if the value of theta is less than θ_{m2zero}, the rear equation contributes to the pressure distribution.

This defines the full pressure distribution over the contact patch, which can then be substituted into the shear stress equation to define the full shear stress distribution.

$$\tau = [c + P * \tan(\phi)] * \left[1 - e^{\left(-\frac{r}{K}\right)[\theta_0 - \theta - (1-i)(\sin(\theta_0) - \sin(\theta))]}\right] \tag{10.}$$

2.2.5. Results

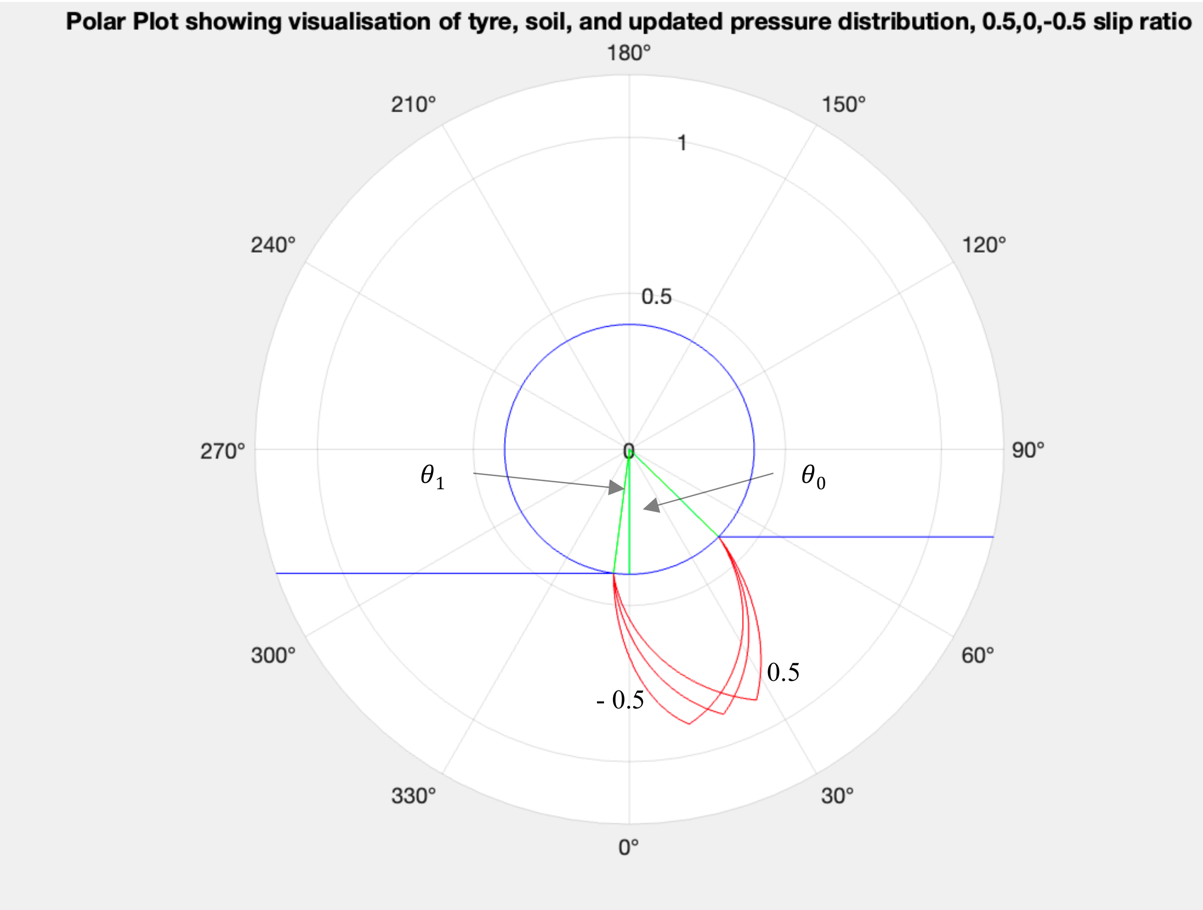


Figure 6, visualising the new pressure ratio, with forward movement of maximum pressure with increasing slip ratio

2.3. Introduction of tread

2.3.1. Concept

Other than the exception of race car tyres, no tyre is a completely smooth perfect circle, with the vast majority of tyres having a tread. Terramechanics tyre treads are typically in the form of large lugs, which aid performance in ways such as increasing traction (even with low loads), and movement of the terrain to avoid digging.

2.3.2. Importance to accuracy

How much tread changes the key factors of vertical load and drawbar pull depends on the size of the tread being modelled. However, the implementation of tread to the model drastically changes how shear stress is calculated.

If a thorough implementation of tread was introduced, the accuracy would be greatly improved, however the method by (Carsten Harnisch, 2005) is a simplified tread model, hence the accuracy of the model is only slightly improved.

2.3.3. Expected results

Tread increases the sinkage over the contact area, meaning an increased pressure distribution, leading to an increased vertical load and drawbar pull.

Shear stress of the soil and tyre will be increased due to an increased pressure distribution, but the minimum value between the two will be used, so the increase may be negligible.

2.3.4. Implementation into model

The same pressure distribution equations (8.) and (9.) listed above are used, over the same total angle range, and with the same point of maximum pressure, but with r substituted for $r + t$, where t is a set value of tread height.

Pressure-sinkage equation region in front of maximum pressure point (θ_{m2zero}):

$$P_{pos} = \left(\frac{k_c}{b} + k_{phi}\right) * ((r + t)^n) * ((\cos(\theta) - \cos(\theta_0))^n \qquad (11.)$$

Pressure-sinkage equation region behind maximum pressure point (θ_{m2zero}):

$$P_{pos} = \left(\frac{k_c}{b} + k_{phi}\right) * ((r + t)^n) * [\cos\left[\theta_0 - \frac{\theta + \theta_1}{\theta_{m2zero} + \theta_1} * (\theta_0 - \theta_{m2zero})\right] - \cos(\theta_0)]^n (12.)$$

Shear stress of soil for positive portion:

$$\tau_{pos\ soil} = [c + P_{pos} * \tan(\phi)] * \left[1 - e^{\left(-\frac{r+t}{K}\right)[\theta_0 - \theta - (1-i)(\sin(\theta_0) - \sin(\theta))]}\right] \qquad (13.)$$

Shear stress of tyre for positive portion:

$$\tau_{pos\ tyre} = P_{pos} * \mu_f \qquad (14.)$$

The maximum shear stress will be the minimum of the two equations above:

$$\tau_{pos\ max} = \min\left[P_{pos} * \mu_f, [c + P_{pos} * \tan(\phi)] * \left[1 - e^{\left(-\frac{r+t}{K}\right)[\theta_0 - \theta - (1-i)(\sin(\theta_0) - \sin(\theta))]}\right]\right] \quad (15.)$$

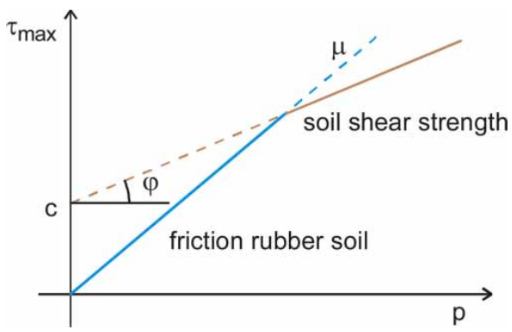


Figure 7, visualisation of equation (14.), (Carsten Harnisch, 2005)

Shear stress of soil for negative portion:

$$\tau_{neg} = c + P * \tan(\phi) \qquad (16.)$$

This defines a base radius pressure and shear stress over the entire contact behaviour, modelled as negative portions of tread, as well as a radius plus tread pressure and shear stress over the entire contact area, modelled as positive portions of tread. Then these total contact area values are multiplied by their retrospective proportions of the contact are.

$$W_{tread} = \left(r * b * 0.6 \left(P_{int} + \tau_{neg_{int}}\right)\right) + \left((r+t) * b * 0.4 \left(P_{pos_{int}} + \tau_{pos\,max_{int}}\right)\right) \quad (17.)$$

P_{int} and τ_{int} are for loops ran to calculate the vertical components of pressure and shear stress over the contact patch.

2.4. Pneumatic tyre characteristics

2.4.1. Concept

The model previously calculated pressure based on Bekker's pressure (or adaptation of Bekker's), which is based on a non-deformable plate at a specific sinkage, modelling the tyre as rigid. However, the real-life situation is a tyre sat on an alloy inflated to a specific inflation pressure, which can be deformed at a certain force.

So, there is a force downwards exerted on the terrain, which at a certain force its current structure won't be able to support, and the terrain will deform. But there is also a force upwards on the tyre, which at a certain force its current structure won't be able to support, and the tyre will deform. So, the magnitude of force it takes the tyre to deform, compared to the magnitude of force it takes the terrain to deform will determine how much the tyre and terrain deform.

2.4.2. Importance to accuracy

On very easily deformable terrains, even a pneumatic tyre won't deform, or its deformation will be negligible, so in this case modelling a pneumatic tyre isn't important to the accuracy of the model. However, on less deformable terrains, a pneumatic tyre will deform substantially, in which case modelling a pneumatic tyre is very important to the accuracy of the model. So, modelling pneumatic tyre behaviour is more important to the validity of the model over a wide range of terrains, as opposed to the accuracy on a single terrain.

2.4.3. Expected results

If at a set entry and exit angle, the shape of the tyre no longer follows a perfect circle, as it has been deflected inwards, this means the values of sinkage along the contact patch will be lower, and so it is expected that pressure values along the distribution are less. Lower values along the pressure distribution, and hence lower values of shear stress, would mean lower values of vertical load and drawbar pull.

2.4.4. Implementation into model

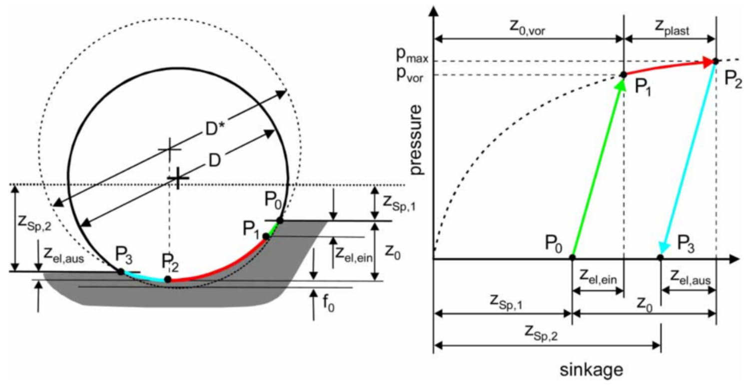


Figure 8, Visualisation of substitute tyre used for modelling pneumatic characteristics (Carsten Harnisch, 2005)

Equation to find distance tyre is compressed:

$$f_{0\,initial} = \frac{W_{tread}}{C_z} \tag{18.}$$

Equation used in iteration to calculate substitute value of radius, (C A Bekakos, 2016):

$$\sqrt{\frac{r^*}{r}} = \sqrt{1+\frac{f_0}{z}} + \sqrt{\frac{f_0}{z}} \tag{19.}$$

The value of r* is now the radius of the first substitute circle. However, visualising figure 8 above, the actual entry and exit points stay the same, with the centre of the substitute circle moving. If the value of r* was just implemented back into the model as the new radius, this would model the substitute circle having a different centre point, which changes the entry and exit angles, where the angle range is crucial to force calculations.

The situation here is two circles with different equations intersecting at two points. The first circle has a centre point (0,0) with radius as the initial radius r, and the substitute circle has an unknown centre point with a radius r*. The coordinates where they intersect are $[rsin(\theta_0), -rcos(\theta_0)]$ and $[-rsin(\theta_1), -rcos(\theta_1)]$, taking θ_1 as positive.

The midpoint of the chord:

$$x_{midpoint} = \frac{(x_1 + x_2)}{2} \tag{20.}$$

$$y_{midpoint} = \frac{(y_1 + y_2)}{2} \tag{21.}$$

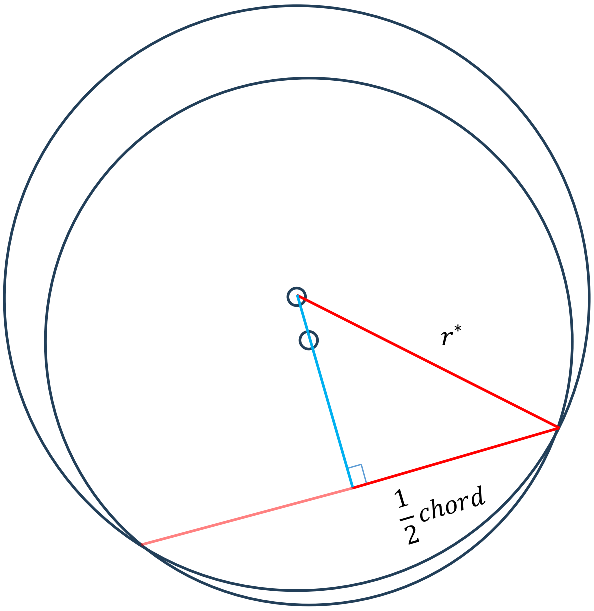


Figure 9, visualising the calculation to find the equation of the substitute circle

The length of the chord:

$$chord_{length} = \sqrt{\left(\frac{(x_2 - x_1)}{2}\right) + \left(\frac{(y_2 - y_1)}{2}\right)^2} \tag{22.}$$

The distance to centre of substitute circle:

$$distance_{centre} = \sqrt{(r^*)^2 - \left(\frac{chord_{length}}{2}\right)^2} \tag{23.}$$

The unit vector direction of the chord:

$$unit_x = \frac{(x_2 - x_1)}{chord_{length}} \tag{24.}$$

$$unit_y = \frac{(y_2 - y_1)}{chord_{length}} \tag{25.}$$

Unit vector perpendicular to chord:

$$perp_x = -unit_y \tag{26.}$$

$$perp_y = unit_x \tag{27.}$$

The position of the centre of the new circle is calculated from taking the coordinates of the midpoint, and summing this to the distance to the centre of the substitute circle by the perpendicular direction to the chord

$$h = x_{midpoint} + distance_{centre} * perp_x \tag{28.}$$

$$k = y_{midpoint} + distance_{centre} * perp_y \tag{29.}$$

This defines the equation of the substitute circle in relation to the origin, which is the centre of the wheel hub.

The distance from the origin to the point on the circumference of the substitute circle (deformed radius) can now be calculated over the total angle range. Then the sinkage can be calculated at each point by taking the vertical component of the distance and subtracting from the vertical distance to the entry point.

When the range of distances from the origin to points on the circumference of the substitute tyre is defined. The process described from equation (11.) onwards is repeated but substituting deformed radius values instead of r values. The contact area is not simply r*b anymore, so the contact area is calculated by a trapezium rule integration of deformed radius*b for the range of angles. This leads to a new W_{tread} value, and now the r* value becomes r in equation (19.), and a new r* value is generated. The iteration is carried out 3 times, as there is negligible change in the radius value after the 3rd value, and this helps to speed up processing time. The result of this series of iterations is an accurate range of sinkages over the contact area, based on the centre of the initial circle (the hub of the wheel). The final value of vertical load is calculated within the iterations, and the final value of drawbar pull is calculated by summing the horizontal components of the final range of pressures and shear stresses.

3. Implementation of tyre model into Simulink

3.1. Walkthrough of Simulink function block

- Takes input of data which is terrain data for a specific soil. Also takes an input of the sinkage and slip ratio.
- Calculates entry angle based on sinkage
- Defines a range of theta values from zero (vertical) to entry angle
- Calculates range of pressures according to the range of angles by subbing theta range into Bekker's pressure equation
- Calculates rear pressure distribution with method in 3.1
- Finds exit angle with method in 3.1
- Creates a new Pressure distribution with a maximum point in front of bottom dead centre with method 3.1
- Creates shear stress distribution using input data, pressure distribution and slip ratio
- All of inner pressure, outer pressure, inner shear stress and outer shear stress vertical components are integrated over the theta range from entry to exit, by using the trapezium rule and for loops.
- Vertical load is calculated by summing these integrations and multiplying by ratio of positive portion to negative portion of tread.
- Vertical load is used to calculate deformation of a pneumatic tyre, which starts a 3-stage iteration for the radius of a substitute tyre, with its corresponding vertical load.
- Drawbar pull is calculated by integrating horizontal components and summing the integrations and multiplying by ratio of positive portion to negative portion of tread.
- Multiplies vertical load and drawbar pull by 1000 to convert from kN to N.
- Outputs vertical load and drawbar pull (can output average radius for improved accuracy but decreased processing time)

3.2. Vertical modelling

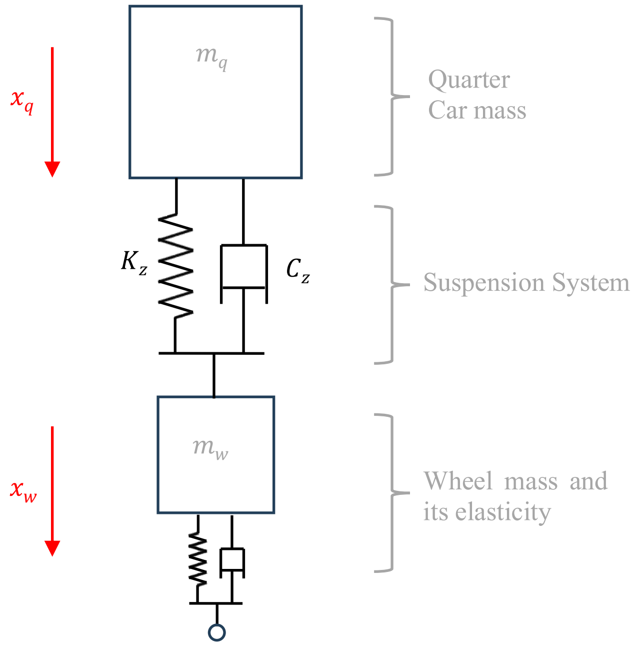


Figure 10, Schematic representation of vertical dynamics in quarter car model

Wheel mass equation of motion:

$$M_w\ddot{x}_w + K_z\left(x_w - x_q\right) + C_z\left(\dot{x}_w + \dot{x}_q\right) - F_z + M_w g \tag{30.}$$

¼ car mass equation of motion:

$$M_q\ddot{x}_q + K_z\left(x_q - x_w\right) + C_z\left(\dot{x}_q - \dot{x}_w\right) = M_q g \tag{31.}$$

3.3. Horizontal modelling

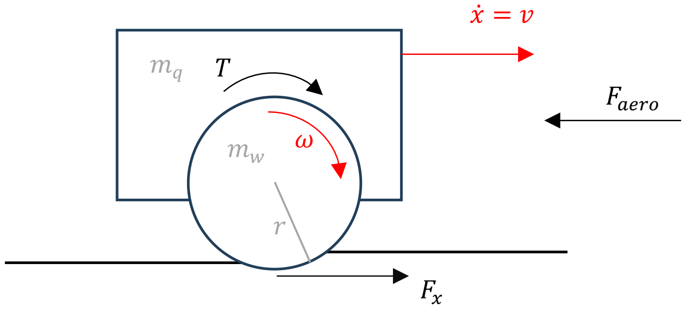


Figure 11, Schematic representation of horizontal dynamics in quarter car model

Linear equation of motion:

$$M_{total}\dot{v} + \frac{1}{2}\rho A C_d v^2 = F_x \tag{32.}$$

Angular equation of motion:

$$I\dot{\omega} = T - F_x r \tag{33.}$$

3.4. Individual modelling

3.4.1. Process of vertical feedback loop

The initial values mean that the weight of the wheel and quarter car will produce a force downwards, leading the wheel to accelerate into the terrain. The distance by which the wheel sinks into the terrain will determine the magnitude of the upwards force produced by the terrain. As the sinkage increases, the reaction force by the terrain increases, decreasing the net force downwards and slowing the acceleration into the soil. When the sinkage is large enough, the reaction force by the ground will be large enough to counter the force of the weight and accelerate the wheel upwards. However, as the wheel's sinkage decreases, the reaction force by the soil decreases, and the weight of the wheel and quarter car produces a net force downwards again. This cycle continues, however the suspension system and wheel have damping, so the peaks of the positive and negative forces are decreased each cycle, until the point where there is no net force and hence no net acceleration upwards or downwards, and an equilibrium is met. The result of running this feedback loop over a set time is a constant sinkage value in the terrain, where at this sinkage the upwards force produced by the terrain is equal to the downwards force produced by the weight of the wheel and quarter car.

3.4.2. Process of horizontal feedback loop

The initial values mean the wheel is moving forwards slowly (to avoid divide by zero error), but with a positive torque applied to it. This results in acceleration and an increase in velocity, however if the velocity becomes very large, the drag force will increase and slow the acceleration down.

3.4.3. results

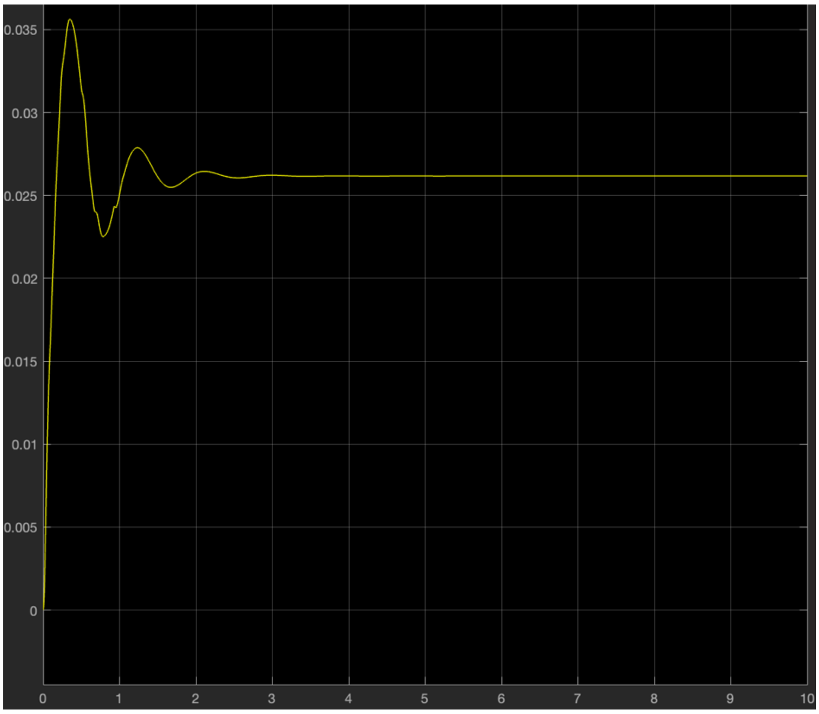


Figure 12, showing damping and eventual equilibrium of sinkage

3.5. Combined modelling

Initially both loops were ran separately, allowing the results to be validated independently. In those cases, a slip value of zero was used on the vertical loop, and F_x was a constant value multiplied by slip ratio for the horizontal loop. When both loops were producing results as expected, the slip output from the horizontal loop was fed into the vertical loop (as slip is a component of shear stress), and the F_x from the vertical loop was fed into the horizontal loop.

How this affects the process for the vertical loop is that the positive slip slightly increases the vertical load (positive reaction force of the terrain), but this is due to the tyre digging and so equilibrium sinkage should be higher.

How this affects the process for the horizontal loop is that the forwards force F_x will be a function of sinkage as well as slip, instead of just slip multiplied by a constant, and is a more accurate value of forwards force.

3.5.1. Initial values

The initial sinkage is set to zero, with an initial velocity of 1 m/s (to avoid divide by zero error) and initial angular velocity of 2.5 rad/s (so slip is initially zero).

3.5.2. Results

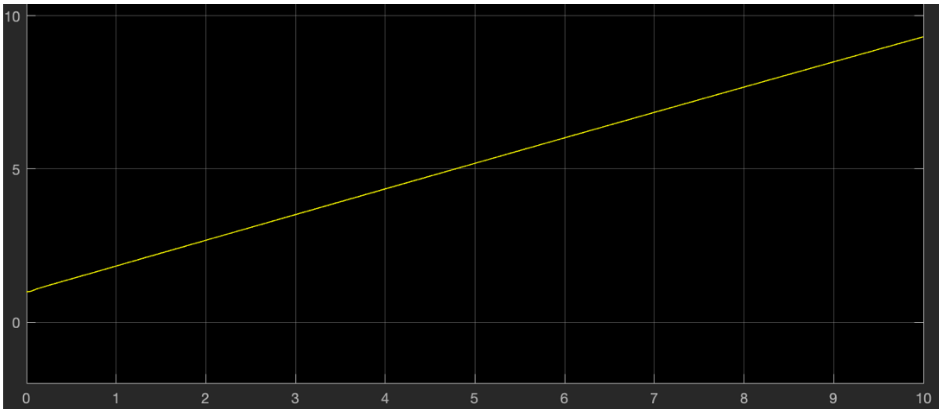


Figure 13, showing a linearly increasing velocity from a constant torque input

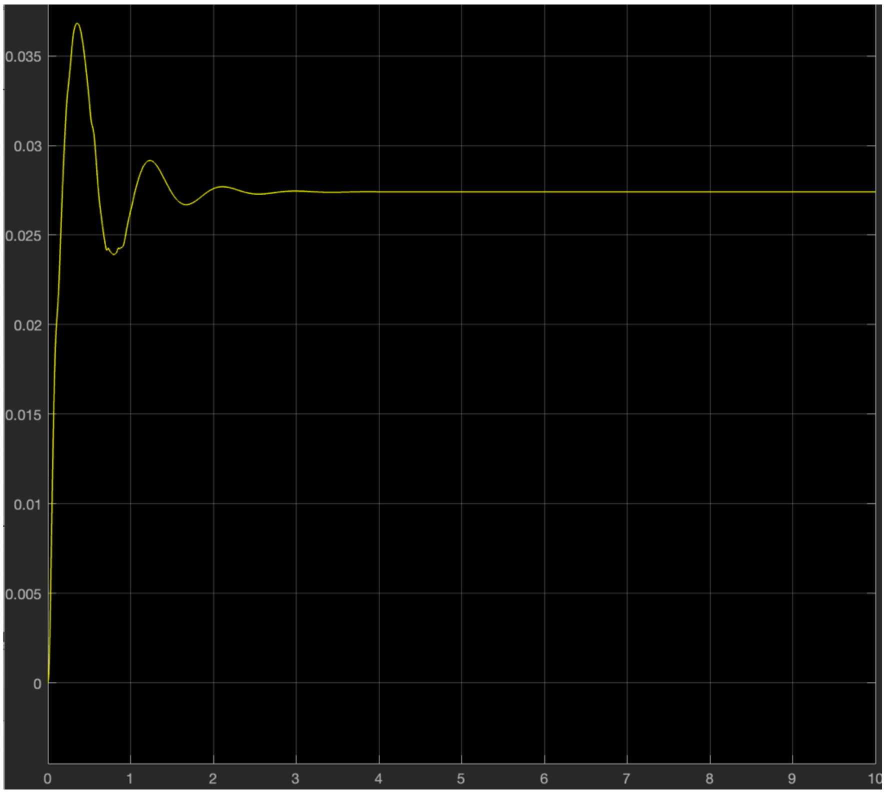


Figure 14, showing damping and eventual equilibrium of sinkage

The equilibrium sinkage is higher with positive values of slip, which was expected.

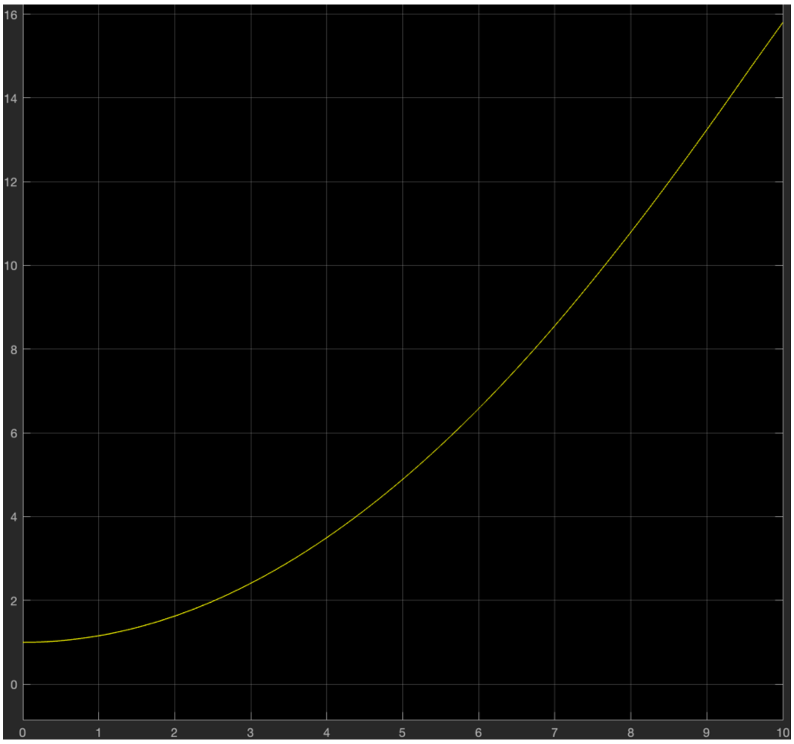


Figure 15, showing initial exponential increase in velocity, that transitions into linear increase due to drag, from an increasing torque

4. Industry challenges

From the literature review conducted in the project preparation report, there was a clear theme where each tyre model or alteration of Bekker's pressure model was due to previous models not being able to adapt to a unique challenge that their research introduced. So, industry challenges were directing the research and engineering solutions.

Although the creation of the tyre model in this research was part of an investigation of tyre-terrain relationships, rather than for aid of a unique industry challenge, research on industry challenges and additional desired performance factors has been conducted. This can also help analyse the capability of the tyre model and ¼ car model produced and point to improvements that are needed to solve current industry challenges.

4.1. Rally

From an interview with rally car driver Dean Mascarenhas, (Krishnan, 2019) states that suspension in rally cars is designed to help drivers take corners, hit bumps and go through ruts at extremely high speeds, listing height clearance and the degree to which the car rolls on turns being important factors affected by suspension systems.

This highlights a challenge where height clearance restrictions need to be met for a rally car to be legitimate, but the damping of the system needs to be optimised for an increase in height or jumps and a decrease in height over ruts, with potential periods of no contact with the terrain. How well the suspension system is set up will greatly affect the speed that is able to be carried over these unique parts of terrains.

(Krishnan, 2019) also mentions how rally cars require air scoops to bring air in to cool the engine. Ensuring the engine doesn't overheat is crucial for a rally car to complete stages, however air scoops increase the drag coefficient of the car, which will reduce the top speed capability of the car. This highlights the challenge of a balance of reliability and performance.

4.2. Agriculture

(V. Shobhan Naik, 2019) states that better fuel efficiency results in less consumption for a specific task, reducing cost and environmental impact.

This presents the challenge that the ability to use certain machinery, as well as crop yields, are based on timing around the weather, but running machinery at its maximum performance constantly increases fuel consumption and wear on parts, increasing running costs and maintenance costs. So, finding the point of maximum power, while still maintaining good efficiency and a lower constant strain on components is crucial.

5. Analysis of modelling types

One clear distinction of different types of modelling is FEA modelling and empirical/semi-empirical modelling. Where FEA can model highly complex tyre shapes and can simulate a unique environment, for example FEA modelling of a Mars rover tyre. In this scenario the long run time of FEA modelling can be used to improve tyre material and design (things that cannot be altered once the mission has started).

However, empirical/semi-empirical models can compute extremely fast, and real time data can be used to adapt things that can be changed whilst moving, such as tyre pressure (tyre stiffness), suspension stiffness, and drag coefficient. This type of modelling will generally be used in well understood environments, where previous experiments have taken place, so there is parameter data for empirical models.

6. Analysis of Simulink model

6.1.1. Analysis based on additions to model

All additions of phenomena to the model follow validated research previously completed, and values for dimensions and other variables are typical terramechanics tyre values. However, no tuning of many constants has been completed, and all values haven't been taken from one specific terramechanics vehicle.

6.1.2. Analysis based on ability to solve/improve industry challenges

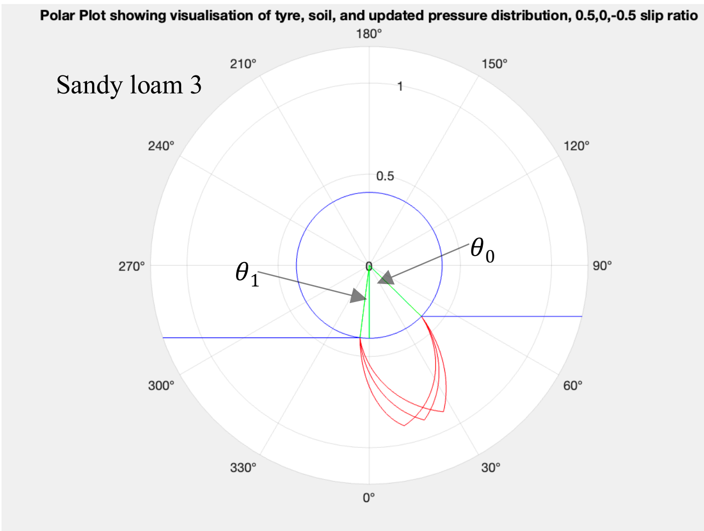
Industry challenges in rally result in suspension height/ground clearance, as well as rolling of the car through corners to be of high importance, and the model doesn't include both of these factors. However, acceleration and velocity, as well as damping of the suspension and drag are incorporated into the model.

Although the model doesn't incorporate fuel consumption and component wear into the model, the torque input could be adapted for a specific engine to relate to RPM of the engine, to ensure the vehicle is in the optimal gear and RPM.

6.1.3. Additional analysis points

All constants such as ¼ car mass, air density, drag coefficient, etc, can be changed for an individual scenario, or easily adapted to be a function of time (for example to model fuel consumption), meaning the model is very versatile. The resolution of angles over entry to exit angle can be increased or decreased, depending on whether pure accuracy or speed of modelling is preferred.
All current cars have traction control, ABS and many other systems that are feedback loop systems, so the technology is available in current cars if this type of modelling was to be introduced for active control in terramechanics modelling. Although more research and additional sensors would likely be required.

7. Summary



Terramechanics is defined by (Wong, 2001) as *'the study of the performance of an off-road vehicle in relation to its operating environment (the terrain)'.* This project began with simplified modelling using pressure–sinkage and shear stress equations, applied over the front portion of a circular tyre profile. Basic off-road tyre dimensions and constant or varying slip ratios were used to explore fundamental relationships between terrain and tyre forces. Terrain return due to elastic unloading was introduced, defining an exit angle and the total angle over the contact area. This allowed for calculation of a forward shifted point of maximum pressure, with the degree of shift being a function of slip. From this a new pressure distribution was defined using an equation provided by (REECE, 1967) – an adaptation of Bekker's equation. This pressure distribution was substituted into shear stress equations to model negative and positive portions of tyre tread. This was still under a rigid tyre assumption, so the vertical load was used to iteratively compute the deflection of a pneumatic tyre, producing a deformed contact patch and varying effective radius. Updated vertical load and drawbar pull were calculated across this arc. The tyre model was implemented in a Simulink-compatible function block and placed in a quarter-car suspension model. Vertical and horizontal feedback loops were created and coupled once independent results were verified, allowing dynamic interaction between slip, torque, sinkage, and suspension response. Final outputs and graphs of velocity and sinkage verified expected behaviour and equilibrium vertical load. The work highlights key force interactions while pointing to future extensions including lateral forces, damping, and secondary effects like fuel efficiency and mechanical wear.

8. Conclusion

As well as both aims of the project, the result of the project is a base function block of code that models lots of tyre phenomena, but which doesn't model various other tyre phenomena. The Simulink code is very easy to adapt, and all of the constants of the model can be changed easily. So, the end result of this research could be the starting point of additional research and an even more complex tyre model.

All variables such as tyre radius, tread height, tread coverage, toque inputs, aerodynamic drag all change the many of key force and performance indicators that are crucial to solving industry challenges. So, the model allows for a specific off-road vehicle to be implemented and its response to torque visualised.

The use of a wide array of empirical data allows for a specific tyre design to be implemented into the model, and key performance indicators found. This could help design a tyre/suspension system that is needed for a specific requirement, without the need for greater empirical relationships to be generated.

The model, with some refinement, could also be used in real time for active control.

The Simulink model works and gives expected results on many of the terrains, however on some terrains the model produces errors where sinkage just constantly increases, as opposed to the damping motion presented in the results. Additional troubleshooting of problems in the function block or Simulink model would be required to overcome these errors.

The first aim was to 'investigate tyre and terrain relationships and how they affect performance factors of an off-road vehicle'. Initially terrain relationships (pressure-sinkage and shear stress-displacement) were investigated in relation to measuring devices at a specific sinkage, with no relation to tyre behaviour. This investigation discovered empirical data and equations that could be used with dimensions and other data from tyres. A library of terrain data was collected at this stage. Rough tyre dimensions of an off-road vehicle were introduced, and information such as slip could be set at a constant value or varied to find relationships. The project preparation report used pressure-sinkage equations and shear stress equations in relation to a set sinkage and radius of tyre, to generate vertical load and drawbar pull values based on the front portion of the tyre. This resulted in the production of a simple tyre model, where many assumptions were made to simplify force prediction calculations, to produce graphs that showed general relationships.

Throughout this report more complex phenomena around the behaviour of a pneumatic tyre with tread on various terrains has been introduced to the model. This resulted in the production of an improved tyre model with greatly improved accuracy. The model was then implemented into a ¼ car model which simulates vertical and forward motion of a ¼ car mass, suspension and tyre system on a specific terrain over a period of time. This is very important as data from vertical model feeds into horizontal model and data from horizontal model feeds into vertical model.

Considering the literature review conducted in the preparation report, analysing the findings from very thorough investigations of tyre and terrain relationships, to build a strong foundational understanding of the subject, the creation and advancement of a tyre model in MATLAB code, which generated graphs to visualise the relationships and changes to the performance factors as new phenomena was introduced, and then implementing this model into Simulink, to generate graphs of performance factors and movement (acceleration, velocity, sinkage and slip) over a period of time, this aim has been met.

The second aim was to 'investigate desired performance factors of certain industries, developing an understanding of unique requirements and challenges.' The literature review in the preparation report started this investigation, highlighting the importance of vertical load and drawbar pull, as well as the various modelling techniques. This gave insight to desired performance factors, but more research was conducted in order to build a deeper understanding. Findings of this additional research were presented, showing lateral forces and damping as additional key factors. As well as bringing to light the secondary effects such as fuel consumption, mechanical wear, engine cooling and physical clearances. An analysis of the two main types of tyre modelling was completed, allowing for the categorisation of the tyre model produced, and its potential use cases.

Research into how ABS, traction control and other feedback loop data systems work could be considered to look to potentially add the tyre model to a cars electrical system to allow for active control. This raises the question of what additional equipment would be needed?

As (Carsten Harnisch, 2005) mentions, the method for modelling tyre tread does not consider the shape of the tread or the distribution of tread over the tyre. Hence, further research could be done for more accurate modelling.

All the terrain data is for homogenous terrain, so it can't model reactions to other obstacles typically in these terrains such as boulders, bumps and ruts.

The orientation of modelling is horizontal and vertical, however in many real-world situations there would be incline and decline, which would greatly change the forces in the horizontal and vertical orientations.

The modelling is with the tyre pointed straight forwards (parallel to direction of motion), however a car is going to be steering throughout a large proportion of its motion. During steering there are also many other factors for example understeer/oversteer and lateral forces.

This tyre model doesn't consider the multipass effect to model the rear tyre, or if another car has driven in front of the one being modelled, so improvements could be made here.

Appendix

	1 terrain name	2 moisture content (%)	3 n	4 kc (kN/m^n+1)	5 kphi (kN/m^n+2)	6 c (kPa)	7 phi
1	'Dry sand'	0	1.1000	0.9900	1.5284e+03	1.0400	28
2	'Sandy loam 1'	15	0.7000	5.2700	1.5150e+03	1.7200	29
3	'Sandy loam 2'	22	0.2000	2.5600	43.1200	1.3800	38
4	'Sandy loam 3'	11	0.9000	52.5300	1.1280e+03	4.8300	20
5	'Sandy loam 4'	23	0.4000	11.4200	808.9600	9.6500	35
6	'Sandy loam 5'	26	0.3000	2.7900	141.1100	13.7900	22
7	'Sandy loam 6'	32	0.5000	0.7700	51.9100	5.1700	11
8	'Clayey soil 1'	38	0.5000	13.1900	692.1500	4.1400	13
9	'Clayey soil 2'	55	0.7000	16.0300	1.2625e+03	2.0700	10
10	'Heavy clay 1'	25	0.1300	12.7000	1.5560e+03	68.9500	34
11	'Heavy clay 2'	40	0.1100	1.8400	103.2700	20.6900	6
12	'Lean clay 1'	22	0.2000	16.4300	1.7247e+03	68.9500	20
13	'Lean clay 2'	32	0.1500	1.5200	119.6100	13.7900	11
14	'LETE sand'	0	0.7900	102	5301	1.3000	31.1000
15	'Upland sandy ...	51	1.1000	74.6000	2080	3.3000	33.7000
16	'Rubicon sand...	43	0.6600	6.9000	752	3.7000	29.8000
17	'North Gower c...	46	0.7300	41.6000	2471	6.1000	26.6000
18	'Grenville loam'	24	1.0100	0.0600	5880	3.1000	29.8000
19	'Snow 1'	-1	1.6000	4.3700	196.7200	1.0300	19.7000
20	'Snow 2'	-1	1.6000	2.4900	245.9000	0.6200	23.2000
21	'Snow 3'	-1	1.4400	10.5500	66.0800	6	20.7000

Figure 16, Library of terrains

```
g = 9.81; %gravity
mq = 500; % 1/4 car mass (kg)
mw = 60; %wheel  mass (kg)
Cz = 2.5e3;
Kz = 30e3;


slip_threshold = 0.3;
K_d = 0.1; %0.1-0.3 kN
K_sinkage = 0.05; %0.05


k_shear = 0.05; % 0.05 standard
% k_const = 80e3 ;
% I = 1;
% T = 1000;

rho = 1.225;
Area = 0.7;
drag_coefficient = 0.4; % 0.4 to 0.45


% max pressure point parameters
c_1 = 0.43;
c_2 = 0.32;

%tread
friction_coefficient = 0.5;

%tyre
r = 0.4;
b = 0.3; % [tyre width (m)]
t = 0.03; %tread
```

Figure 17, Values of variables

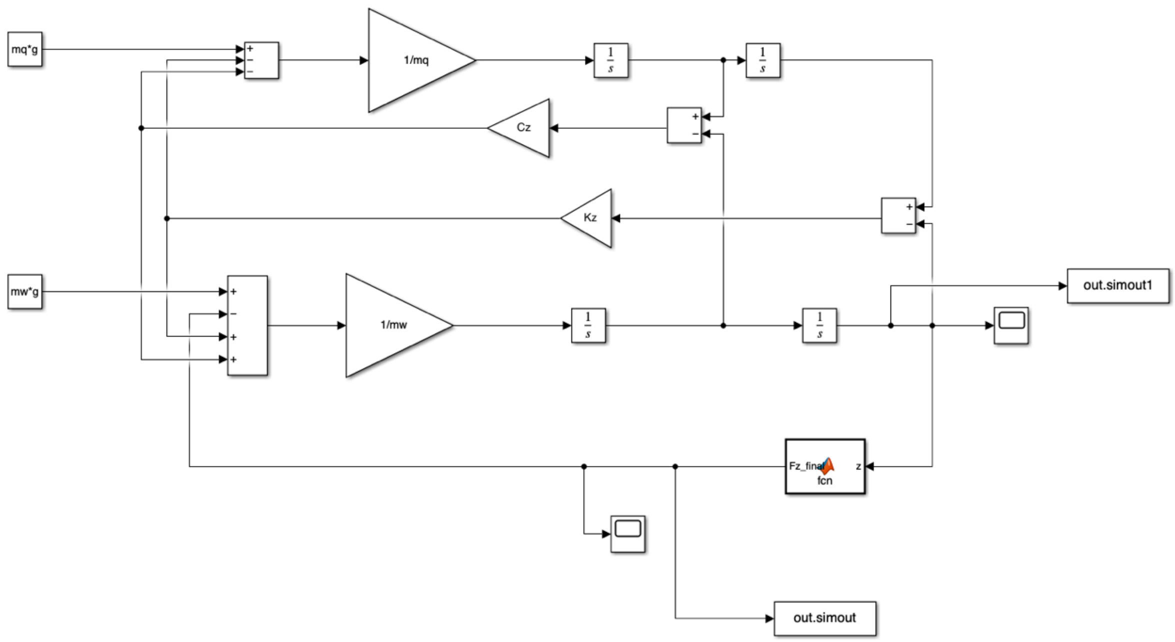


Figure 18, Individual vertical loop

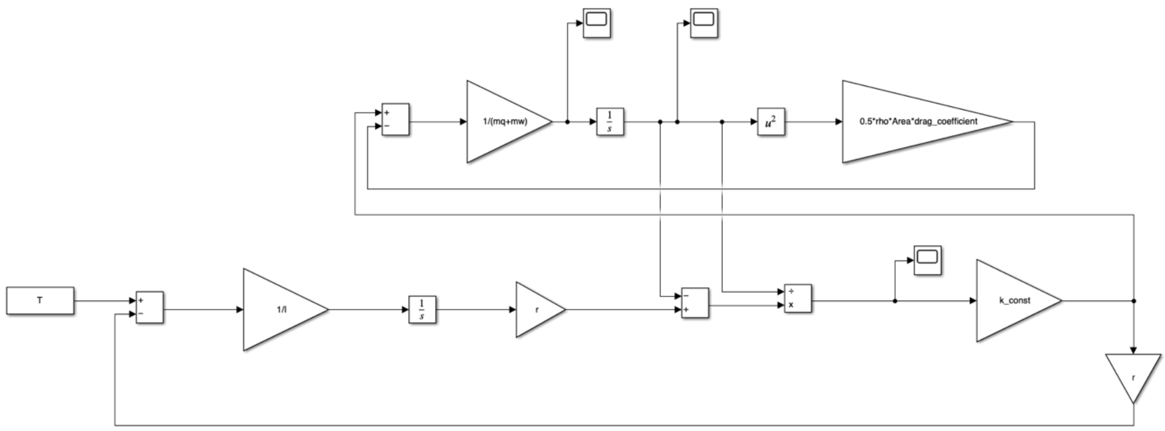


Figure 19, Individual horizontal loop

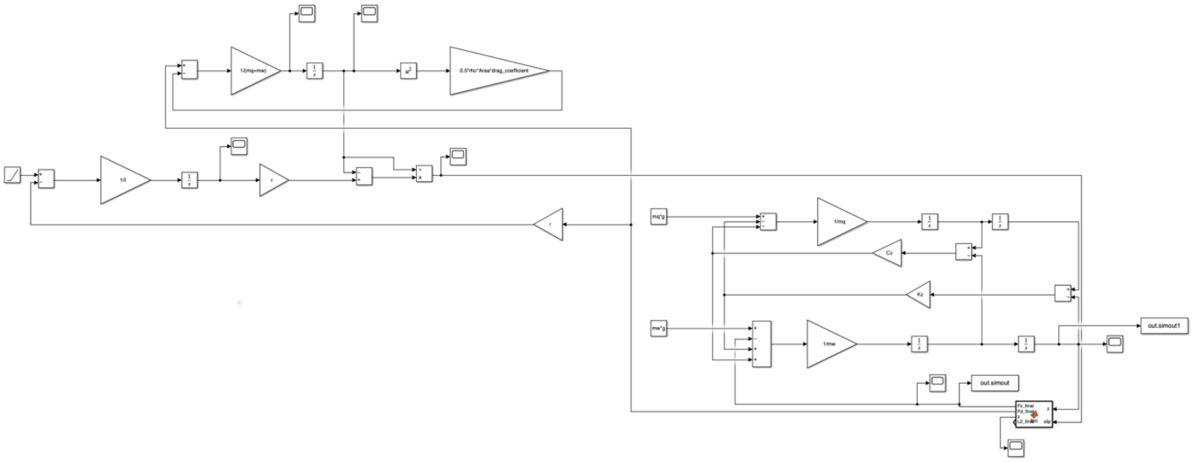


Figure 20, showing combined loop

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Project Meeting Log

<table>
<tr>
<td>Meeting: 1 Date: 17/02/2025
Review of actions from meeting:

Received feedback on project preparation report.

Discussed progress made on terrain return, as well as an alternative way to model return which is more accurate.

Talked about point of maximum pressure and how Wong-Reece equation could solve this.</td>
<td>**Agreed actions for next meeting:**

Look at chapter 2.4 of Wong’s book for explanation and equations to the alternative way to model return

Look at Wong-Reece paper for pressure equation (calculate entry and exit angle, then sub these into the equation to find point of maximum pressure).

Agreed date/time of next meeting: 24/02/2025</td>
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<tr>
<td>Meeting: 2 Date: 24/02/2025
Review of actions from meeting:

Looked at Wong-Reece paper and implemented equations. Produced results on polar plot and got feedback on this in meeting.

Looked into alternative way to calculate return angle but had questions about it. This was discussed in meeting and found that it was working correctly, and the aim was to find the angle where pressure was zero.

Also discussed both pneumatic tyres and tyre tread.</td>
<td>**Agreed actions for next meeting:**

Adapt polar plot by non-dimensionalizing pressure and adding to radius value.

Look at Harnisch’s paper for methods to add pneumatic modelling and tread.

Agreed date/time of next meeting: 17/03/2025</td>
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<td colspan="2">Signed Student: ___ C.Toone _______________ Supervisor: _____ ______</td>
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<tr>
<td>Meeting: 3 Date: 17/03/2025
Review of actions from meeting:

Got feedback on updated polar plots.

Talked about finding equilibrium of radius value in relation to pneumatic tyres.

Discussed order of modelling tread and pneumatic modelling.

Discussed ¼ car modelling (vertical modelling)</td>
<td>**Agreed actions for next meeting:**

Implement tyre tread modelling in the code before pneumatic tyre modelling, instead of after.

Get the radius equilibrium part of the code working.

Begin to implement this code into a ¼ car model

Agreed date/time of next meeting: 24/03/2025</td>
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<td>Meeting: 4 Date: 24/03/2025
Review of actions from meeting:

Got feedback into implementation into ¼ car vertical model. The model was working correctly but extremely slow due to lots of internal and external functions.

Discussed ¼ car modelling (horizontal modelling)</td>
<td>Agreed actions for next meeting:

Go through the code and ensure all the code is done inside the Simulink function and reduce as many internal functions as possible. Reduce resolution if it keeps accuracy the same.

Go through a similar process to creating the vertical loop in Simulink, but for horizontal motion as discussed.

Agreed date/time of next meeting:</td>
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